Materials and Structures, 1994, **27**, 222–228

Determination of the moduli of elasticity of rocks. Comparison of the ultrasonic velocity and mechanical resonance frequency methods with direct static methods

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In the present investigation, dynamic methods for the determination of moduli of elasticity were compared with direct static methods. The dynamic moduli of rocks, such as Young's modulus (E) and Poisson's ratio (ν) were determined, using both mechanical resonance frequency and classic P and S wave ultrasonic velocity techniques. For this purpose rock samples from different areas of France, covering a wide range of velocity values, were used. The mechanical resonance frequencies were investigated using a Grindo-Sonic machine while the P and S wave ultrasonic velocities were measured using a Pundit ultrasonic machine, connected to an oscilloscope. The static moduli were determined using deformation gauges. Statistical interpretation of the test results indicated significant correlation between these dynamic and static methods. Accordingly, the above non-destructive dynamic methods are suitable for the determination of static moduli of elasticity.

1. INTRODUCTION

Moduli of elasticity, used to express the deformability of rocks, may be obtained by dynamic methods in addition to static compression or shear tests. Dynamic moduli of elasticity are obtained by the rapid application of stress to the sample. In the present investigation, two different dynamic methods were used to provide data comparable with those obtained using ordinary static methods. The first dynamic method uses P and S wave ultrasonic velocity measurements, along core specimens, while the second uses the excitation and detection of mechanical resonance frequencies in small cylindrical or prismatic specimens. The same rocks were also tested to determine their static moduli of elasticity. A direct compressional technique is used for this purpose. Small deformation gauges, attached both horizontally and vertically to the specimen axis, provide the deformation data. The test results were compared statistically with each other, using regression methods to verify the static moduli using dynamic, non-destructive techniques. These dynamic methods (instead of the direct static ones) are simple and preserve the specimens.

2. STATIC ELASTIC MODULI

Deformation data may be obtained for specimens undergoing strength tests and used to calculate the static moduli of elasticity of intact rock. The modulus of elasticity (E, or Young's modulus) and Poisson's ratio (ν) are the most commonly used. The modulus of elasticity, which is derived from a form of Hooke's law, is obtained from the applied axial compressive stresses and the resulting axial strains. Poisson's ratio is calculated from the axial and diametral strains resulting from applied axial compressive stresses.

They are both useful in estimating the elastic response of intact rock to compression from *in situ* construction and post-construction stresses. Abutment stresses in a dam or those exerted against the rock by water-pressure tunnels are examples of post-construction stresses. The values for the modulus E may be obtained from stress–strain diagrams. Between the average modulus, tangent modulus and secant modulus, referred to in the literature, the last mentioned is the most common, predicting the maximum elastic deformation that would occur at 50% of ultimate strength [1].

3. DYNAMIC MODULI OF ELASTICITY

3.1 Ultrasonic velocity tests

The modulus of elasticity E_d, and Poisson's ratio ν_d may be obtained by dynamic methods, as well as by static compression tests. One common dynamic method for determining moduli is to subject the rock sample to compression and shear wave pulses. Compression and shear wave transducers are attached to the ends of the core specimen for this purpose. Wave velocity is calculated from the travel time of the pulse through the specimen. Samples may be loaded to approximately field conditions because both P and S wave responses increase

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with compression. Typically, the dynamic modulus of elasticity is greater than the static, because the response of the specimen to the very short duration strain and low stress level is essentially purely elastic [2]. Ultrasonic velocity is not only an indication of the moduli; it is also a very good index for rock quality classification and weathering determination [3-5].

3.2 Mechanical resonance frequencies

The procedure consists of exciting a specimen by means of a light external mechanical impulse and of the analysis of the transient natural vibration during the subsequent free relaxation. This excitation is applied in such a way as to favour the desired vibrational mode. A pinpoint transducer is used to pick up the mechanical vibration [6].

Specimens can be excited easily into flexural or torsional modes in order to obtain the modulus E and the Poisson's ratio, according to the following relationships [7, 8], where f_1, f_2 are the resonance frequencies into torsional or flexural modes, h, D are the thickness and diameter of the specimen, C is a constant depending on the dimensions of the specimen, the vibrational mode and the Poisson's ratio, f_n is the resonance frequency of order n, and d is the density.

$$\nu = g[(f_1/f_2), (h/D)]$$

$$E = Cdf_n^2$$

4. THE ROCK MATERIAL

In the present investigation, eight different rock samples from France, covering a wide range of properties, were used (Fig. 1). The rocks studied are as follows.

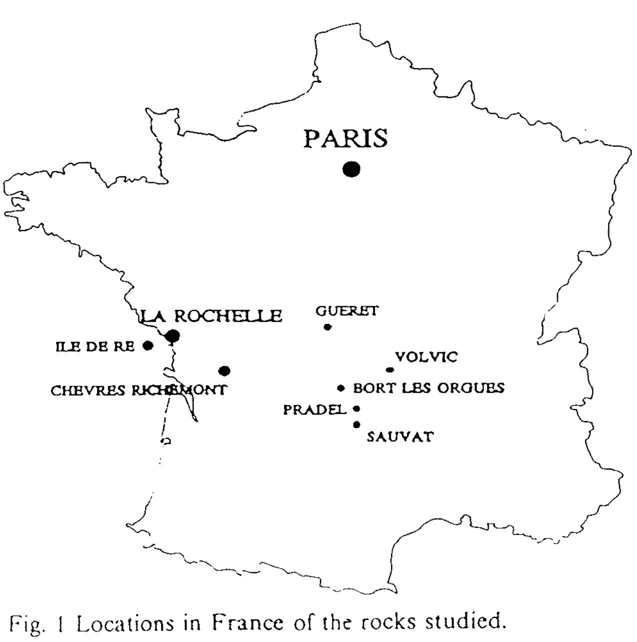

Fig. 1 Locations in France of the rocks studied.

Limestone from Ile de Ré: Fine grained, calcitic limestone of Jurassic age. It is homogeneous and massive, without closely spaced cracks. The colour is light yellow. No important physico-mechanical anisotropy was observed.
Gypsum from Chevres Richemont: A calcium saccharoidal sulphate of rose colour and massive structure. No important physico-mechanical anisotropy was observed.
Basalts from Sauvat and Pradel: Fine grained mafic volcanic rocks of dark green colour, composed of Ca-plagioclases and augite. No important physico-mechanical anisotropy was observed.
Granite from Gueret: Coarse grained, massive and compact plutonic acid rock, composed of quartz, K feldspars and biotite. No important physico-mechanical anisotropy was observed.
Phonolite from Bort les Orgues: Fine grained acid rocks, containing dominant nepheline and both K and Na feldspars. No important physico-mechanical anisotropy was observed.
Andesite from Volvic: Fine grained, intermediate, volcanic, igneous rock, composed of microcrystalline plagioclases, biotite and augite. It exhibits important physico-mechanical anisotropy, due to the formation process. The test values obtained dynamically are statistically significant when compared with the static ones, but even so, they should be viewed with a question mark.

5. EXPERIMENTAL METHODS

All the tests were carried out in La Rochelle, using the laboratory facilities of the Laboratoire de Construction Civile et Maritime, Physique et Mécanique des Matériaux de Construction, Université de Poitiers, La Rochelle, France).

5.1 Preparation of the specimens

Rock specimens of two different dimensions were prepared, according to our test requirements. The static loading and ultrasonic velocity tests were applied on 49 mm dia. × 100 mm height core samples. Specimens were prepared using a core drilling machine. The mechanical resonance frequencies were measured on 49 mm dia. × 5 mm thick cylindrical rods. Densities were measured on both core samples and cylindrical rods. Experiments involving static loading and ultrasonic velocities were applied on the same specimens; static measurements were made after the dynamic ones. The rods used for determining resonance frequencies and the specimens used for the previous experiments were cut from the same cylindrical samples, which obviously were longer than 100 mm before cutting.

5.2 Ultrasonic velocities

Ultrasonic velocities were measured as compression V_p and shear (V_s according to the French specification AFNOR NF B 10505. Both compression and shear wave

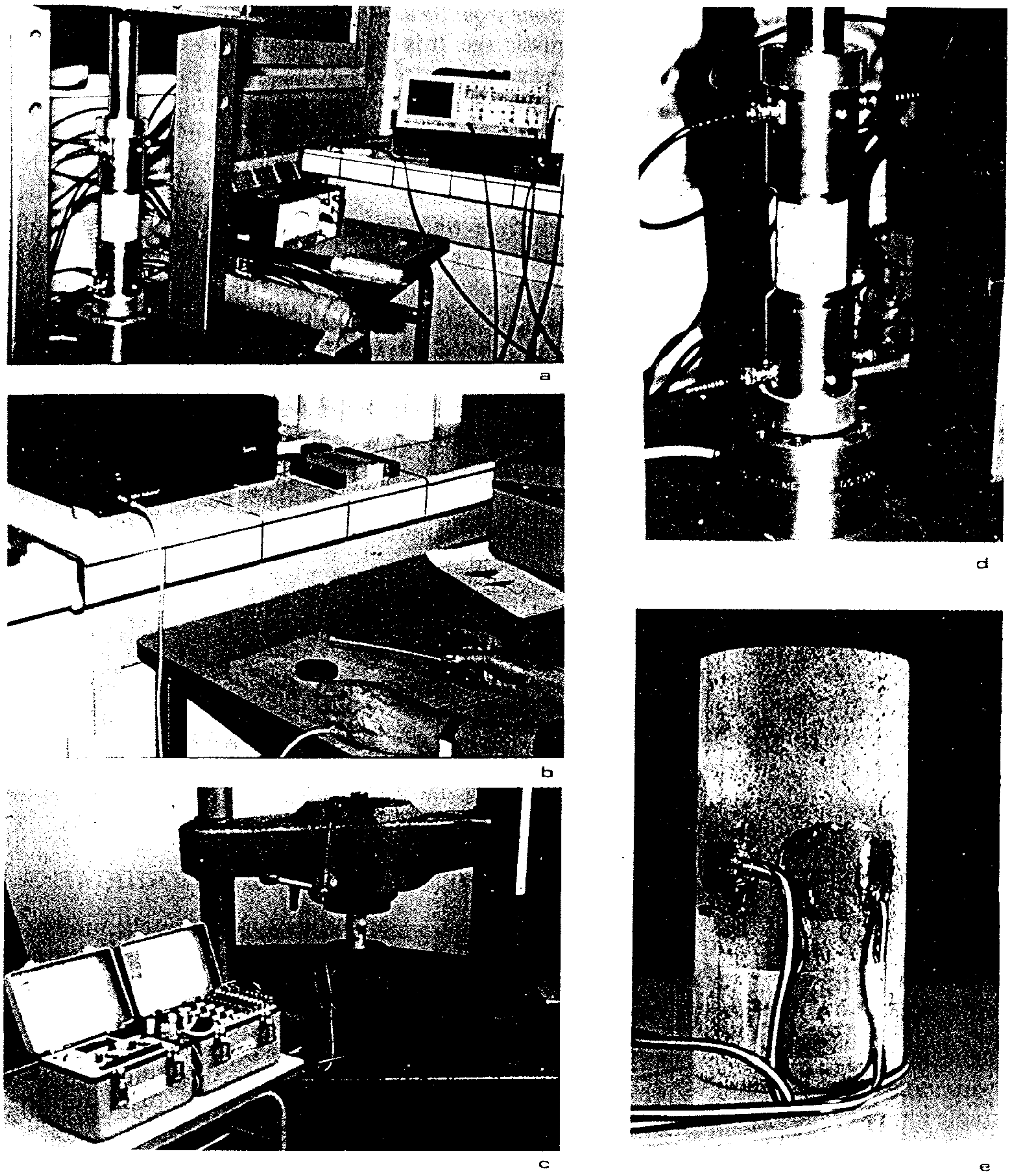

Fig. 2 Experimental techniques: (a) P and S wave measurement system; (b) Grindo-Sonic methods; (c) strain measurements; (d) P and S wave transducers; (e) deformation gauges.

measurements were made using a Pundit velocimeter. Specific transducers that measure P and S wave velocities were supplied by Laboratoire de Construction Civile et Maritime, Université de Poitiers, La Rochelle to CNS Electronics for this purpose; S waves can be measured simultaneously in two perpendicular directions (Fig. 2(a,d)). The travel time of the 300 kHz source pulse was measured using a Gould 1602 numerical oscilloscope. Measurements were applied along the axis of the core samples. A 30 mm dia. × 0.04 mm thick lead foil disc was used as a coupling medium, to improve the acoustic contact between the sample and the transducers. The instrument was calibrated with aluminium standards. Thickness and travel time corrections were calculated by linear regression between the actual and the measured times.

Shear wave velocities were measured in two perpendicular directions (S_1, S_2) to detect possible physico-mechanical anisotropy. S_1 and S_2 values were found to be more or less similar, so a mean value of S_1 and S_2 wave velocities was used in our interpretation. Only the andesite from Volvic exhibited significant anisotropy, according to these tests, but for this sample a mean S wave value was also used. Measured values of P and S wave velocities (V_p, V_s) are given in Table 1, and the values of the dynamic moduli (E_d, ν_d), calculated from the above velocities, are given in Table 2.

5.3 Mechanical resonance frequencies

Measurements were performed using a Grindo-Sonic instrument. Tests were carried out on 49 mm dia. × 5 mm

Table 1 Pundit P and S wave measured velocities (V_p, V_s) and velocities (V_{pg}, V_{sg}) calculated from Grindo-Sonic data. Densities (d) are also given

Rock	V_p (m s^{-1})	V_s (m s^{-1})	V_{pg} (m s^{-1})	V_{sg} (m s^{-1})	d (g cm^{-3})
Limestone Ré	3500	2117	3254	1972	2.20
Gypsum Rose	5117	2281	5569	2530	2.31
Andesite Volvic	3810	2198	3754	2377	2.20
Basalt Sauvat	6353	3688	6423	3965	3.00
Basalt Pradel 1	6410	3708	6412	3750	3.01
Basalt Pradel 2	6542	3813	6520	3848	3.05
Granite Gueret	5194	3192	5088	3194	2.67
Phonolite	5258	3187	7103	2784	2.58

Table 2 Static (E_{st}, ν_{st}), Grindo-Sonic (E_{dg}, ν_{dg}) and dynamic (E_d, ν_d) moduli of elasticity calculated from Pundit data

Rock	E_{st} (GPa)	E_d (GPa)	E_{dg} (GPa)	ν_{st}	ν_d	ν_{dg}
Limestone Ré	19.88	24.70	20.70	0.177	0.184	0.21
Gypsum Rose	36.10	33.08	40.50	0.336	0.376	0.37
Andesite Volvic	28.72	26.58	29.33	0.183	0.251	0.18
Basalt Sauvat	101.83	101.66	105.00	0.246	0.246	0.24
Basalt Pradel 1	110.63	103.34	105.00	0.246	0.249	0.24
Basalt Pradel 2	114.37	110.21	110.00	0.244	0.243	0.24
Granite Gueret	63.98	65.11	64.00	0.176	0.196	0.175
Phonolite	56.50	63.39	56.00	0.289	0.210	0.410

thick cylindrical rods. The principles of this method were described previously. To excite the response, a light and elastic tap was given in the centre or on the side of the specimen, depending on our decision to obtain a longitudinal, flexural or torsional vibration. To detect the resulting vibration and to convert it into electrical signals, a hand-held piezoelectric detector was used, in contact with the test sample (Fig. 2(b)). For modulus E_{dg} and Poisson's ratio ν_{dg} determination, flexural and torsional vibrational frequencies were measured. Torsional measurements were made in two directions and a mean value was used for the calculations of the moduli. Calculations were performed using a specific computer program, containing all the necessary correction factors. Measured values of the dynamic moduli E_{dg} and ν_{dg} are given in Table 2, and the velocity values V_{pg} and V_{sg}, calculated from the above moduli of elasticity are given in Table 1.

5.4 Static moduli of elasticity

The static Young's modulus E_{st} and Poisson's ratio ν_{st} were measured using specific deformation gauges, of universal type (Vishay, EA and CEA; 120 Ω; coefficient of temperature auto-compensation $11 \times 10^{-6}\,°\mathrm{C}^{-1}$), attached to the cylindrical surface of the specimens (Fig. 2(c, e)). The static Young's modulus is measured as the secant modulus of elasticity (deformation under loading at 50% of the strength, Fig. 2(c)). Tests were performed on the samples used previously for ultrasonic velocity tests, and the results are given in Table 2.

5.5 Density

Density values d were obtained according to the specification ASTM C 97–47, by dividing the dry weight by the total volume of the specimens, after drying for more than 48 h at 80°C. Test results are given in Table 1.

5.6 Conversion formulas used between ultrasonic velocities and dynamic moduli [1]

These equations are valid for more or less isotropic media. In anisotropic rocks, like the andesite from Volvic, they could be valid only for the direction of the measurement. In the case of the andesite from Volvic, S wave velocities were measured, in different directions, perpendicularly to the main axis of the core sample; mean values were eventually used, for uniformity of test results.

Young's modulus:

$$E_d = kdV_s^2\,\frac{3V_p^2 - 4V_s^2}{V_p^2 - V_s^2}$$

Poisson's ratio:

$$\nu_d = \frac{V_p^2 - 2V_s^2}{2(V_p^2 - V_s^2)}$$

P wave velocity:

$$V_{pg} = \sqrt{\frac{E_{dg}(1 - \nu_{dg})}{d(1 + \nu_{dg})(1 - 2\nu_{dg})}}$$

Table 3 Regression relations between the methods used, together with correlation coefficients (r) and standard deviations

X/Y	Regression	Correlation coefficient r	Standard deviation Y
Pundit dynamic/static elasticity modulus	$E_{st} = -3.16 + 1.05E_d$	0.994	38.02
Grindo-Sonic dynamic/static elasticity modulus	$E_{st} = -3.12 + 1.05E_{dg}$	0.997	38.02
Pundit dynamic/static Poisson's ratio	$\nu_{st} = 0.063 + 0.71\nu_d$	0.737	0.057
Grindo-Sonic dynamic/static Poisson's ratio	$\nu_{st} = 0.029 + 0.85\nu_{dg}$	0.962	0.057
Pundit/Grindo-sonic P wave velocities	$V_{pg} = -270.85 + 1.05V_p$	0.988	1334
Pundit/Grindo-Sonic S wave velocities	$V_{sg} = 45.72 + 1.01V_s$	0.982	801.9
Grindo-Sonic/Pundit elasticity modulus	$E_d = 0.83 + 0.98E_{dg}$	0.992	35.79
Pundit P wave/static elasticity modulus	$E_{st} = 3.02\,e^{0.00055V_p}$	0.970	38.02

S wave velocity:

$$V_{sg} = \sqrt{\frac{E_{dg}}{2d(1+\nu_{dg})}}$$

k is a constant.

6. RESULTS AND DISCUSSION

Test results were evaluated statistically to determine the relationships between the methods used. These relationships were expressed mathematically (Table 3) and by regression diagrams. Correlation coefficients and standard deviations were also determined. The correlations were verified for their significance, at the 99% level, for $n - 2 = 6$, by calculation of the following equation, using the Student's tables (t is the Student value, r is the correlation coefficient, n is the number of samples).

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

For the test results given a highly significant correlation (99% level) between the static and the dynamic moduli was observed. According to Tables 1 and 2, the test values are more or less similar. The values for the anisotropic andesite from Volvic are also reasonable, because mean values of the static and dynamic tests were used. Nevertheless, test values of this sample should be considered with a question mark. The resonance frequencies in phonolite show an important difference between the static and ultrasonic velocity methods. This difference arises because phonolite is extremely sensitive to acoustic sounds or other external vibrations, so the present method is very difficult to apply to this rock type.

According to the correlation diagrams of Figs 3 and 4, the regression slope between the static and dynamic values is about 1. So, the static modulus of elasticity is calculated constantly about 3 points lower than the dynamic modulus, using either the Pundit or Grindo-Sonic method. For Poisson's ratio, a similarly significant correlation is observed between the static and dynamic methods (Figs 5 and 6).

Correlation diagrams between the velocities as well as between the moduli of the two dynamic methods showed a significant positive correlation. The equation slopes for both velocities and Young's moduli are about 1. Therefore, the calculated P wave velocities of the resonance frequency method may be considered

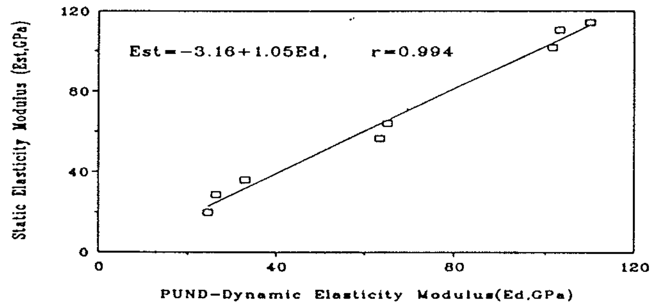

Fig. 3 Correlation diagram of Pundit dynamic versus static moduli of elasticity.

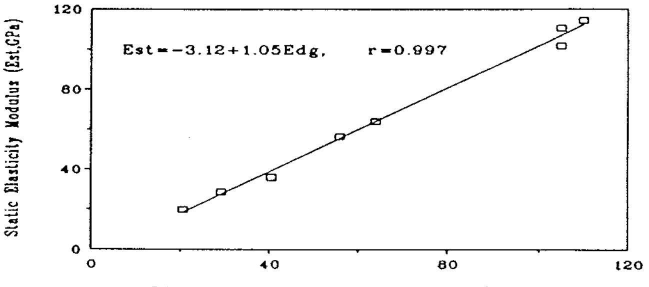

Fig. 4 Correlation diagram of Grindo-Sonic dynamic versus static moduli of elasticity.

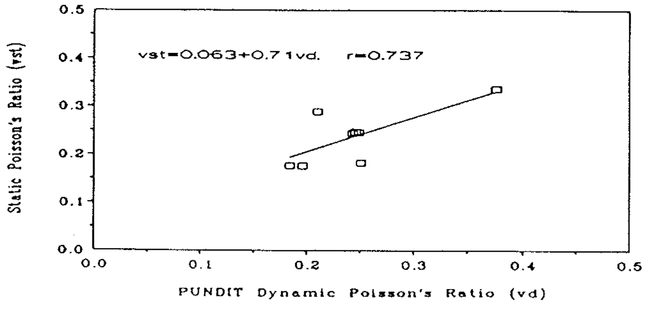

Fig. 5 Correlation diagram of Pundit dynamic versus static Poisson's ratios.

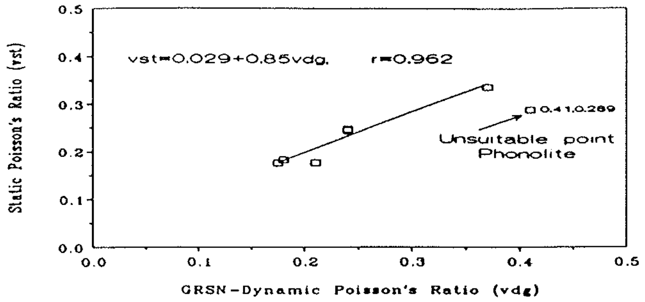

Fig. 6 Correlation diagram of Grindo-Sonic dynamic versus static Poisson's ratios.

consistently lower than the P wave velocities measured by the Pundit method (Fig. 7). On the other hand, the S wave velocity is consistently higher using the resonance frequency method compared with Pundit (Fig. 8). The two dynamic Young's moduli are similar, providing consistently higher values using the Pundit method instead of the Grindo-Sonic (Fig. 9). Values of Poisson's ratio from the above dynamic methods when compared with each other also show a significant correlation (Fig. 10), and a significant correlation using an exponential regression is observed between the static Young's modulus and the P wave velocities measured directly using the Pundit method (Fig. 11). According to our interpretation of our results, the static moduli of elasticity can be determined using dynamic methods. The consistent difference observed between the static and dynamic methods provides confirmation.

The equations determined are compared in Fig. 12 with those in recent literature [9, 10]. The difference, of about 25%, observed between the equations proposed and those of the literature is probably due to the different type of static Young's modulus measured; in [9] the type of modulus *E* is not defined and in [10] the modulus *E* is a tangent modulus.

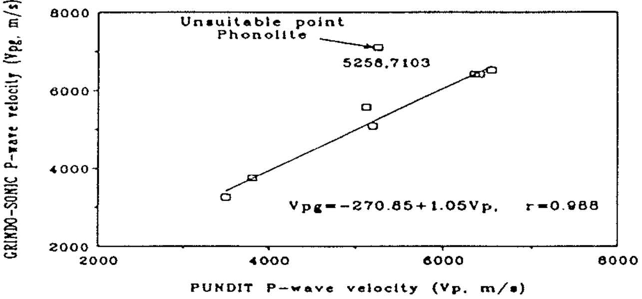

Fig. 7 Correlation diagram of Pundit versus Grindo-Sonic P-wave velocities.

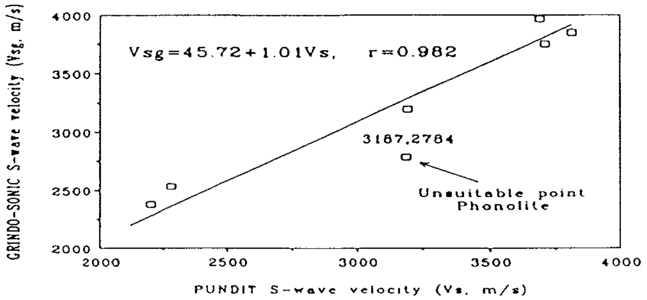

Fig. 8 Correlation diagram of Pundit versus Grindo-Sonic S wave velocities.

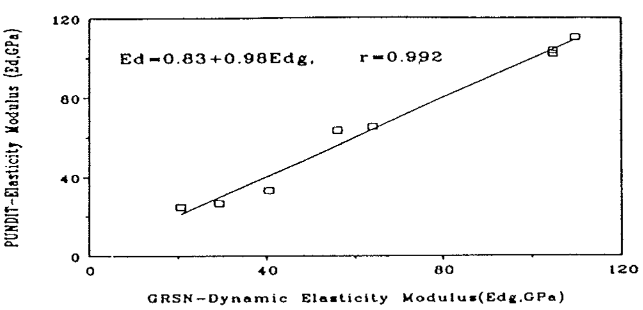

Fig. 9 Correlation diagram of Grindo-Sonic versus Pundit dynamic moduli of elasticity.

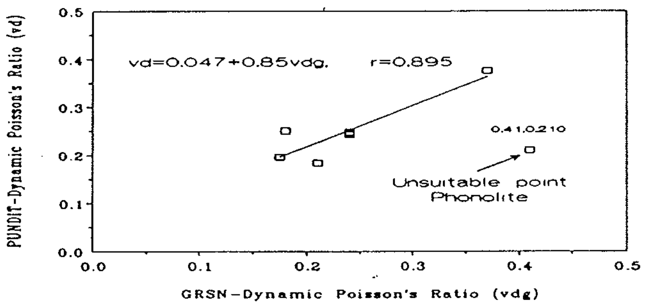

Fig. 10 Correlation diagram of Grindo-Sonic versus Pundit Poisson's ratios.

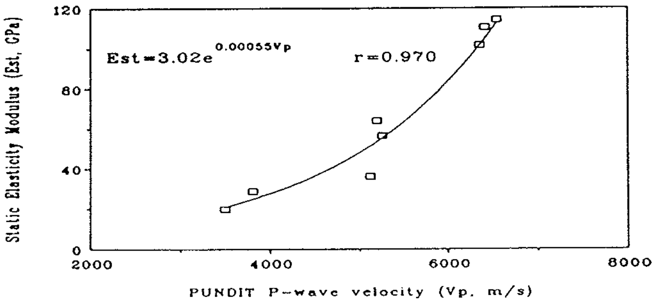

Fig. 11 Correlation diagram of Pundit P wave velocity versus static moduli of elasticity.

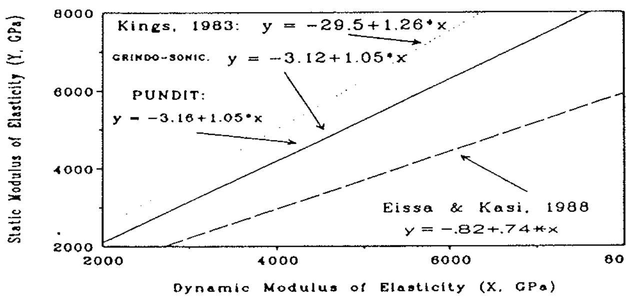

Fig. 12 Comparison of the equations determined with those in the literature.

Comparing the principles of the two dynamic methods, we would expect the test results obtained by the resonance frequency method to be more reasonable than those obtained by the ultrasonic velocity method. The reason is that the first are based directly on the vibrational frequency of the sample, in addition to the density, instead of the travel time of a pulse of constant frequency as measured by the second method. However, test results from both methods present the same relationship with those of the static method, so either of them can be used for a quick estimation of the static moduli of elasticity. It is worth noting that the elastic moduli of the rocks are not only an expression of the compressibility and mechanical strength but also a function of the susceptibility to stone salt decay [11].

7. CONCLUSIONS

In our investigation, two dynamic non-destructive methods were used for estimating static moduli of elasticity of rocks. The first method involves P and S wave ultrasonic velocity determination while the second uses mechanical resonance frequency detection. The results were compared and the correlation was good. The two methods provided results that were significantly comparable with those obtained by the static method. A consistent difference noted between the static and dynamic values underlines our observation. The correlations were determined by linear and exponential regressions as given in Table 3.

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RESUME

Détermination des modules d'élasticité des roches. Comparaison des méthodes de la vitesse du son et de la fréquence de résonance avec la méthode statique

Dans la présente étude, les méthodes de détermination du module d'élasticité dynamique sont comparées aux méthodes de calcul statique direct. Les modules d'élasticité dynamique des roches tels que le module de Young (E) et le coefficient de Poisson sont déterminés en utilisant à la fois une méthode de résonance en fréquence et la méthode classique de mesure de la vitesse des ondes P et S. Pour cette recherche, différentes roches de France, couvrant un large éventail de vitesses des ondes, ont été choisies, la méthode de résonance en fréquence a été établie en utilisant un appareil Grindo-Sonic, tandis que pour la mesure des ondes P et S, nous avons utilisé un Pundit et les lectures ont été effectuées sur un oscilloscope. Le module d'élasticité statique a été déterminé en utilisant des jauges de contrainte. L'interprétation statistique des essais fournit une bonne corrélation entre les méthodes dynamiques et statiques utilisées. En accord avec nos recherches, le module d'élasticité statique peut être déterminé avec une bonne approximation en utilisant des méthodes dynamiques non destructives.